Optimal Termination, Credit Restriction, and Business Cycles

Yu Jin
Iowa State University
Incomplete Draft
September 17, 2009

Abstract

In a model of equilibrium business cycle, financial intermediation plays an important role to generate fluctuations. Lendings are dynamic contracts between intermediaries and firms, and credit restrictions are used as an incentive device. Intermediaries cut off firm’s credit line to inspire the high effort. This will generate fluctuations without fundamental productivity changes.

1 Introduction

The condition of firm credit history is an important determinant of aggregate economic activity. For example, Smith and Wang (2006) found that a dynamic loan contract in which low reported output caused the borrower’s credit position to deteriorate was very effective at mitigating agent’s shirking in a general equilibrium model. They also found that optimal provision of intertemporal incentives led to increasing consumption inequality over time within generational cohorts.

In this paper, we study the role of firm credit history in an equilibrium business cycle model. Financial intermediaries restrict firm credit in some state in order to inspire high effort. Credit restriction (or termination) is an instrument of incentive in a long-term relationship between firm and intermediary. The incentive effect of termination was first studied in credit and labor markets by Stiglitz and Weiss (1983), and applied to executive contracts by Spear and Wang (2005). Stiglitz and Weiss solved a special case where the agent is risk-neutral and limited liability constraint is not binding. They did not give a general optimal termination condition. Spear and Wang solved the general optimal termination condition in an executive compensation model. We apply their model in the credit market where an entrepreneur is risk-neutral and has a binding limited liability constraint.

In a dynamic setting, borrowing constraints are important in generating persistent fluctuations and amplifying business cycles (Scheinkman and Weiss, 1986, Kiyotaki and Moore, 1997). There have been efforts at modelling explicitly borrowing constraints and financial intermediation in
equilibrium business cycle models, e.g. Williamson, 1987, Bernanke and Gertler, 1989, .... They used a "costly state verification" model to study the asymmetric information between firms and intermediaries. In Williamson, the auditing cost is different for each firm ex-ante, and in Bernanke and Gertler, the exogenous difference is project's investment cost. During the recession, inefficient firms will be driven out of the market. The relationship between the firm and the intermediary is static in their models. While we introduce a dynamic loan contract with moral hazard so that firms are ex-ante identical. Firms face different credit constraints based on their different prior performances.

We show that a financial intermediary, who can not observe the entrepreneur's activity directly, faces a trade-off between inducing effort and giving up a limited liability rent to the firm or cutting off the credit line of the firm. Credit is cut off when, due to past low output, the firm's continuation payoff is so close to its minimum payoff that the bank can no longer motivate a higher effort. When, due to search frictions, some of these banks fail to find alternative borrowers, there will be a decline of aggregate economic activity.

In this context, shocks that affect entrepreneur's incentive to exert high effort can lead to cycles. A specific pre-cycle payoff shock is introduced in our model. The shock is mean-preserving so that without financial frictions there are no fluctuations in the model economy. For simplicity, we assume the shock does not affect the output of the firm if the entrepreneur chooses high effort, but affects the risk of the project if low effort is chosen. Put another way, assume there are two different projects with the outcome being either high or low. One is a good project in the sense that the expected return is higher and the risk shock has no effect on this project, e.g. to produce food. But entrepreneur need to pay an additional cost (Food production may be a boring job.). Another project does not need this additional cost, e.g. to make a movie. However, it is a bad project in the sense the expected return is lower, and a risk shock does affect the project. Whenever there is a shock, the return of the this project is riskier (A good movie can still succeed in recession, such as Star War in the 1970's.). With limited liability constraint, the entrepreneur prefers a riskier project which will make the intermediary hard to motivate the entrepreneur to choose the good project. Another example would be that people speculated in risky investment (e.g. real estate, stocks, or tulip bulb) instead of producing before crashes as we recurrently observed in the book of Kindleberger and Aliber (2005).

2 The Model Setup

Time is discrete and the horizon is infinite: \( t = 1, 2, \ldots \). The economy consists of a sequence of overlapping generations with intragenerational heterogeneity. The measure of individuals in the economy is equal to one. Each newly born individual becomes either a worker or an entrepreneur with probability \( 1 - \alpha \) and \( \alpha \) respectively.

Workers born at time \( t \) live two periods. We assume there is one old agent alive for each young agent born. As in Williamson (1987), workers are endowed with \( h \) units of labor time in the first period of life, and they maximize the expected utility

\[
U(c_t, l_t, c_{t+1}) = u(c_t, l_t) + E_t[c_{t+1}],
\]
where $c_t$ is consumption at time $t$ and $l_t$ is leisure consumed at time $t$. $E_t[.]$ denotes the expectation at time $t$. Assume that first-period consumption and leisure are normal goods, that is,

$$u_{12} - u_{11} > 0,$$
$$u_{12} - u_{22} > 0.$$

Each unit of labor time produces one unit of consumption good.

Entrepreneurs, like workers, live two periods. The number of births and the number of deaths are equal in each period, so the total measure of entrepreneurs in the economy, $a$, is invariant. Entrepreneurs are risk neutral.

### 2.1 Entrepreneur

The basic model of financial contracting is introduced by Innes (1990) in a static setup. Each entrepreneur is risk-neutral and is born with zero wealth. Each has access to a technology that produces a stochastic output in the end of each period of the life time. The profit is produced with two inputs: (1) one unit of capital, and (2) entrepreneurial effort $e_t \in \mathbb{E}$. Given effort $e_t$, the profit $\theta_t \in \Theta$ in period $t$ is distributed with density $\pi(\theta_t | e_t)$. In order to finance for his investment, the entrepreneur must borrow in the credit market.

Assume the entrepreneur’s period utility function, $H(c, e)$, is additively separable. Due to the assumption of risk neutrality, the function takes the form

$$H(c, e) = c - v(e),$$

where $c \in \mathbb{C}$ is consumption, $v(.)$ measures the disutility of effort. We also assume that an entrepreneur has a reservation utility equal to $w_0 = 0$ in each period.

### 2.2 Financial Intermediation

Financial intermediaries (banks) arise as institutions of delegated monitoring (Diamond, 1984). There is a unit measure of risk-neutral banks that gather deposits from the workers. While the bank cannot observe the effort $e_t$, it observes the entrepreneur’s ex-post output, $\theta_t$. The distribution of the output depends on the effort so that the bank can infer the entrepreneur’s effort choice from the output. Therefore, the financial contract should specify an payoff function, $B(\theta_t)$, to the bank depending on the observed outcome in order to motivate the effort.

Limited liability constrains the contract in two ways: the entrepreneur cannot repay more than the profit, $B(\theta_t) \leq \theta_t$; and the bank’s liability is limited to its investment in the firm, $0 \leq B(\theta_t)$. The latter assumption simply for making the entrepreneur’s expected utility be bounded above so that the entrepreneur’s optimization problem is well defined. Innes (1990) proved that with monotone likelihood ratio property (MLRP) and a constraint that the payoff function is nondecreasing in output, the standard debt contract (SDC) is the optimal contract form.

Our model takes as given that there is an ongoing relationship between an entrepreneur and the bank. The bank lends to the firm and promises the firm a certain level of expected utility.
At the end of the period, the bank chooses to renew the loan or cut off the initial entrepreneur’s credit line and replace him with a new firm. For convenience, we assume that once the credit line is cut off, no bank will finance this entrepreneur’s project.

3 A Dynamic Loan Contract

For simplicity, suppose from now on that profit $t$ can take only two values, or $\Theta \equiv \{\theta_1, \theta_2\}$, with $\theta_1 < \theta_2$. And there are two different effort levels, $e \in E \equiv \{e_1, e_2\}$ with $e_1 < e_2$. Normalize the disutility of low effort to zero, $v(e_1) = 0$, and $v(e_2) = \psi > 0$. Let $\pi_i(e_j) = \Pr\{\theta_t = \theta_i | e_j\}$ be the probability that the project yields output $\theta_i$ if effort $e_j$, with $\sum_{i=1,2} \pi_i(e_j) = 1$. We assume $0 \leq \pi_2(e_1) < \pi_2(e_2) < 1$.

**Definition 1** The functions $\{\pi_i(e)\}_{i=1,2}$ are said to satisfy the monotone likelihood ratio property (MLRP) if $e_1 < e_2$ implies that $\frac{\pi_1(e_1)}{\pi_1(e_2)} > \frac{\pi_2(e_1)}{\pi_2(e_2)}$.

It is easy to check that our model satisfies the MLRP. We also assume that the entrepreneur’s payoff is nondecreasing in his output. So in each period $t$, the standard debt contract is the optimal contract form. Or let $B_i = B(\theta_i)$ be the bank’s payment in the static optimal contract when $\theta_t = \theta_i$. According to the result in Innes (1990), the payment $B_1$ in the low output state equals the full output $\theta_1$, while the payment $B_2$ in the high output state equals some $R > 0$, where $R$ is the lending rate.

We solve for the dynamic loan contract by backwards induction. Following Spear and Srivastava (1987) and others, we use the promised utility to the entrepreneur as the state variable. We omit the time subscript without confusing. Suppose that the bank delivers a promised utility equal to $w \in \Phi \equiv [0, \bar{w}]$ to a firm at the beginning of the second period, where $\Phi$ is the set of feasible utilities. The bank’s second-period value function $V(w)$ is given by

$$V(w) = \max\{V_1(w), V_r(w)\}. \tag{1}$$

Here $V_1(w)$ is the bank’s value function conditional on lending the money to the firm

$$V_1(w) = \max_{\{B_i\}} \sum_{i=1,2} \pi_i(e)B_i - r, \tag{Q1}$$

where $r$ is the deposit rate, or the bank’s cost of collecting funds. Subject to the limited liability constraint, $0 \leq B_i \leq \theta_i$, for $i = 1, 2$, feasibility constraint, $w \in \Phi \equiv [0, \bar{w}]$, the promise-keeping constraint,

$$\sum_{i=1,2} \pi_i(e)c_i - v(e) = w,$$

where $c_i = \theta_i - B_i$ is the entrepreneur’s payoff when output is $\theta_i$, and the incentive compatibility constraint,

$$e \in \arg \max_{\{e'\}} \sum_{i=1,2} \pi_i(e')c_i - v(e'), \forall e' \in E.$$
Conditional on credit cut-off and replacement, the bank’s value function $V_r(w)$ is

$$V_r(w) = \rho \max \{ V_r(w') | w' \geq 0 \} - w,$$

where $\rho$ measures the probability of finding a replacement.

The long-term contract $\sigma$ offered at the beginning of first period of life time takes the following form:

$$\sigma = \{ e, \{ p_{ik}, (B_{ik}, w_{ik}) \}_{i \in \{1,2\}, k \in \{l,r\}} \}$$

where $e$ is the required effort level, $p_{ik}$ is the probability of the entrepreneur’s credit status being $k$ in period 2 if his realized output in period 1 was $\theta_i$. In addition, at the end of period 1, according to the contract, the entrepreneur pays $B_{ik}$ to the bank and the bank promises $w_{ik}$ to the firm conditional on the credit status being $k$ in period 2 if the realization of the output in period 1 was $\theta_i$.

The following problem characterizes the optimal contract that promises utility exactly equal to $x$ to the entrepreneur at the beginning of the first period of his life:

$$\max_{\sigma} \sum_{i=1,2} \pi_i(e) \sum_{k=l,r} p_{ik} [B_{ik} + V(w_{ik})] - r,$$

subject to the limited liability constraint, $0 \leq B_{ik} \leq \theta_i$, the other feasibility constraints,

$$0 \leq p_{ik} \leq 1, \sum_{k=l,r} p_{ik} = 1, w_{ik} \in \Phi,$$

for all $i \in \{1,2\}, k \in \{l,r\}$, the promise-keeping constraint,

$$\sum_{i=1,2} \pi_i(e) \sum_{k=l,r} p_{ik} (c_{ik} + w_{ik}) - v(e) = x,$$

where $c_{ik} = \theta_i - B_{ik}$ is entrepreneur’s payoff conditional on the credit status being $k$ in period 2 if the realization of the output in period 1 was $\theta_i$, and the incentive constraint,

$$e \in \arg \max_{\{e'\}} \sum_{i=1,2} \pi_i(e') \sum_{k=l,r} p_{ik}(c_{ik} + w_{ik}) - v(e'),$$

for all $e' \in \mathbb{E}$.

4 Credit Cut-off as an Incentive Device

Spear and Wang (2005) have proved that when the agent’s promised utility is too low to support the desired effort, termination occurs as an incentive device in a executive compensation model. The similar result holds here. Let $\bar{\theta}(e_j)$ denote the expected return of the project with effort level $e_j$. 

5
\[ \bar{\theta}(e_j) = \sum_{i=1,2} \pi_i(e_j) \theta_i. \]

Assume \( \bar{\theta}(e_1) \leq r \) and \( \bar{\theta}(e_2) - \psi > r \), where \( r \) is the deposit rate, so that only the high effort is socially efficient ex-ante, and it is never optimal to implement the low effort.

The bank promises the firm \( w \in \Phi \equiv [0, \bar{w}] \). The promise utility \( w \) is great or equal to 0, otherwise the firm can walk away. Consider the second period contract \( \{ B_i \} \) for an entrepreneur born at period \( t \). Omit the time subscript without confusing, the entrepreneur’s consumption \( c_i = \theta_i - B_i \), for \( i = 1, 2 \). The contract implements high effort, \( e = e_2 \), and the bank promises expected utility equal to \( w \) to the worker. So the following promise-keeping and incentive constraints hold:

\[
(1 - \pi_2(e_2))c_1 + \pi_2(e_2)c_2 - \psi = w, \\
(1 - \pi_2(e_2))c_1 + \pi_2(e_2)c_2 - \psi \geq (1 - \pi_2(e_1))c_1 + \pi_2(e_1)c_2.
\]

The incentive constraint requires that

\[
c_2 - c_1 \geq \frac{\psi}{\pi_2(e_2) - \pi_2(e_1)}.
\]

(3)

Given the debt contract form and the promise-keeping constraint, the threshold expected utility \( w \) to implement high effort \( e_2 \) is

\[
w = \frac{\pi_2(e_1)\psi}{\pi_2(e_2) - \pi_2(e_1)}.
\]

(4)

Thus, if \( w \geq w \) then \( e^* = e_2 \) is implemented, and the optimal consumption is

\[
c_i^*(w) = \begin{cases} 0 & \text{if } i = 1 \\ \frac{w+\psi}{\pi_2(e_2)} & \text{if } i = 2 \end{cases}.
\]

On the other hand, if \( w < w \) then high effort cannot be implemented and the optimal consumption is

\[
c_i^*(w) = \begin{cases} 0 & \text{if } i = 1 \\ \frac{w}{\pi_2(e_1)} & \text{if } i = 2 \end{cases}.
\]

So the value function for the bank conditional on lending the money to the firm is

\[
V_l(w) = \begin{cases} \bar{\theta}(e_2) - \psi - r - w, & \text{if } w \geq w \\ \bar{\theta}(e_1) - r - w, & \text{if } w < w \end{cases}.
\]

(5)

However, it is never optimal to implement the low effort by our assumption. So the bank pays \( w \) to cut off the credit to the firm, and find a new project in the market. The value function conditional on credit cut-off and replacement is

\[
V_r(w) = \rho(\bar{\theta}(e_2) - \psi - r - w) - w.
\]

(6)
Lemma 2 The bank’s value function is

\[
V(w|w, r) = \begin{cases} 
\overline{\theta}(e_2) - \psi - r - w, & \text{if } w \geq w \\
\rho(\overline{\theta}(e_2) - \psi - r - w) - w, & \text{if } w < w
\end{cases}
\]

This tell us that termination is a necessary punishing device if the contract must make the worker sufficiently poor in the second period. To determine the optimal long-term loan contract implement the high e¤ort, the following promise-keeping and incentive constraints hold:

\[
\sum_{i=1,2} \pi_i(e_2) \sum_{k=l,r} p_{ik}(c_{ik} + w_{ik}) - \psi = x,
\]

\[
\sum_{i=1,2} \pi_i(e_2) \sum_{k=l,r} p_{ik}(c_{ik} + w_{ik}) - \psi \geq \sum_{i=1,2} \pi_i(e_1) \sum_{k=l,r} p_{ik}(c_{ik} + w_{ik}).
\]

Proposition 3 The bank’s optimal termination policy associated with a promise to deliver expected utility equal to \( x \) is

\[p^*_2(x) = 0\]

and

\[p^*_1(x) = \begin{cases} 
1, & 0 \leq x \leq w \\
2 - \frac{x}{w}, & w < x \leq 2w \\
0, & 2w < x \leq \overline{w}
\end{cases}\]

Termination is a decreasing function of the entrepreneur’s initial promised utility \( x \). Financial intermediation is carried out at each period \( t \) by a large number of intermediaries who each write large numbers of loan contracts with the entrepreneurs. So we can measure the existing firms in each period by law of large numbers (LLN),

\[\Pi(w) = \frac{\alpha}{2} \left[ 1 - \pi_1(e_2)p^*_1(x) \right] + \frac{\alpha}{2}.
\]

The first part of the right hand side is the entrepreneurs in their second period of life time, and the second part is the newborn entrepreneurs.

Corollary 4 The threshold expected utility \( w \) increases as \( \psi \), \( \pi_2(e_1) \) increases, but decreases as \( \pi_2(e_2) \) increases.

Corollary 5 The measure of the existing firms in each period, \( \Pi(w) \), decreases as \( \psi \), \( \pi_2(e_1) \) increases, but increases as \( \pi_2(e_2) \) increases.


5 Shocks and Business Cycles

In the model, the preference, technology, and population are identical in each period $t$. Therefore the model will not produce cycles that are driven by fluctuations in fundamentals. Stochastic shocks can be introduced into the intertemporal production technology as follows. Let $S_t \in \mathbb{S} \equiv \{1, 2\}$ denote the state of the world at time $t$. The state at time $t$ becomes known to all after time $t$ decisions are made and before generation $t + 1$ agents are born.

In states 1 and 2, the features of the economy are identical to those specified in previous sections, except that in state 2 the investment projects of the entrepreneurs are riskier in the sense that if the entrepreneur exert low effort, he has a chance $\pi'_2(e_1) < \pi_2(e_1)$ to get $\theta'_2 > \theta_2$, with the following condition hold

$$(1 - \pi'_2(e_1))\theta_1 + \pi'_2(e_1)\theta'_2 = (1 - \pi_2(e_1))\theta_1 + \pi_2(e_1)\theta_2.$$ 

So the shock in state 2 is mean preserving to the state 1 and will not produce cycles in the absence of financial frictions.

5.1 Equilibrium

To close the model, we calculate the supply of the funds. Workers born at time $t$ solve the following problem:

$$\max_{\{s_t, h_t\}} u(c_t, h_t) + E_t[c_{t+1}]$$

subject to the budget constraints,

$$c_t + s_t = h - l_t,$$
$$c_{t+1} = r_t s_t,$$

and the resource constraint and nonnegative constraints,

$$0 \leq l_t \leq h,$$
$$c_t, c_{t+1}, s_t \geq 0.$$ 

Here $s_t$ is savings by a representative worker born at time $t$, and $r_t$ is the deposit rate faced by this worker.

Let the state $S_t = j, \forall j$ at time $t$. The following first-order conditions determine a solution under the assumption of an interior solution with $s^j > 0$:

$$u_1(h - l^j - s^j, l^j) = u_2(h - l^j - s^j, l^j)$$

$$u_1(h - l^j - s^j, l^j) = r^j.$$ 

Here, $u_i(\cdot, \cdot)$ denotes the partial derivative with respect to the $i$th argument of $u(\cdot, \cdot)$.

In the credit market, we simply assume banks cannot find replacement after they cut off the credit of an entrepreneur, or $\rho = 0$. The value of a loan contract given promised utility, $w$, is
\( V(w|w^j, r^j) \), which is non-increasing in \( w^j \), decreasing in \( r^j \). Financial intermediaries choose the optimal value
\[
V^*(w^j, r^j) = \max_{\{w\}} V(w|w^j, r^j).
\]
By envelop theorem, \( V^*(w^j, r^j) \) is non-increasing in \( w^j \), decreasing in \( r^j \). The competition among the intermediaries will make the value equal to zero in both states
\[
V^*(w^j, r^j) = 0. \tag{9}
\]
And the quantity of loans is equal to the quantity of deposits
\[
\frac{1 - \alpha}{2} s^j = \Pi^j(w^j). \tag{10}
\]

**Definition 6** Given \( \{p^j_{ir}\}_{i, j \in S} \) from the financial intermediary’s optimal loan contract \( \sigma \), the equilibrium in state \( j \in S \) is \( \{s, l, w^j, r^j\}_{j \in S} \) solved by equations (7)-(10).

### 5.2 Comparative Static Analysis

To show how aggregate variable depend on state \( S_t \), we can carry out a comparative static analysis about the stationary equilibrium. Assume that the model economy is in state 1. By equations (7)-(10), we can solve the equilibrium \( \{s^1, l^1, w^1, r^1\} \). After an unexpected shock, the model economy changes to state 2. Let \( \Delta \pi = \pi'_2(e_1) - \pi_2(e_1) \) be the change of probability of high profit with low effort. Total differentiating equations (7)-(10), we get
\[
(A + B) \frac{\Delta l}{\Delta \pi} + \frac{\Delta s}{\Delta \pi} = 0 \tag{11}
\]
\[
A \frac{\Delta l}{\Delta \pi} - u_{11} \frac{\Delta s}{\Delta \pi} - \frac{\Delta r}{\Delta \pi} = 0 \tag{12}
\]
\[
V_1^* \frac{\Delta w}{\Delta \pi} + V_2^* \frac{\Delta r}{\Delta \pi} = 0 \tag{13}
\]
\[
\frac{1 - \alpha}{2} \frac{\Delta s}{\Delta \pi} - \frac{\Delta \Pi(w) \Delta w}{\Delta \pi} = 0 \tag{14}
\]
where \( A = u_{12} - u_{11}, B = u_{12} - u_{22} \).

First, from (13) we find there is an opposite relationship between change of the threshold of cut-off and the deposit rate:
\[
\frac{\Delta r}{\Delta \pi} = -\frac{V_1^* \Delta w}{V_2^* \Delta \pi} < 0.
\]
Since the threshold expected utility \( w \) increases as \( \pi_2(e_1) \) increases, \( \frac{\Delta w}{\Delta \pi} > 0 \), we predict that the deposit rate will decrease. This is because that in order to promise a high utility to entrepreneur, the financial intermediaries must low the cost of the fund to keep zero profit given the entrepreneurs are ex-ante identical. This is different with the prediction of Williamson (1987) where market rate increase with the shock since high auditing cost firm will be drive out the credit market.
Two other observations from the equations are

$$\frac{\Delta s}{\Delta \pi} = 2 \frac{\Delta \Pi(w) \Delta w}{1 - \alpha} \frac{\Delta w}{\Delta \pi} < 0,$$

$$\frac{\Delta l}{\Delta \pi} = -\frac{A}{A + B} \frac{\Delta s}{\Delta \pi} > 0.$$  

So after the shock, we predict that savings and employment are dropping, which are commonly observed during the recession.

6 Conclusion

In this paper, we construct an equilibrium business cycle model in which financial intermediation plays an important role. Loan contracts with a termination term are long-term relationship between financial intermediaries and entrepreneurs. Due to information asymmetry, intermediaries can not observe the efforts of the entrepreneurs. Credit restrictions are termination of contracts to inspirit the high efforts of the entrepreneurs.

When a specific shock comes, intermediaries will cut off the credit line of the firms often. This itself will generate aggregate fluctuations without the change of economic fundamental. A comparative static analysis shows that a shock increases the threshold expected utility will decrease the market rate. Therefore, workers will lower the savings and labor supply. These are consistent with the observation of stylized facts in recession.

References


