Private International Debt with Risk of Repudiation

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The risk of repudiation plays a central role in determining the size of international capital flows. In this paper I compare a centralized arrangement for international debt, where only governments borrow and lend internationally, with a decentralized arrangement, where individual borrowers have access to international capital markets. I show that a centralized setup allows more international risk sharing and higher welfare than a decentralized setup. That is, there is a positive role for government regulation of international borrowing.

I. Introduction

The risk of repudiation plays a central role in determining the size of international capital flows. With few exceptions the existing literature on repudiation models the borrowing country as a single entity, presumably a government, that makes all borrowing and default decisions. In fact, however, individuals do borrow and lend internationally unless capital controls are present.

This paper compares outcomes under centralized and decentralized arrangements for international borrowing and default decisions. That

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is, it studies the effect of government regulation of international capital flows. I show that a centralized arrangement (capital controls) allows more risk sharing than a decentralized one.

In the centralized setup, governments make borrowing as well as default decisions. If a government decides to default, the country as a whole will be excluded from future international borrowing and lending. In contrast, in the decentralized setup, private agents decide how much to borrow abroad and whether to repay debt. I assume that individuals who default on international debt are banned from international capital markets but can still trade in domestic markets. One can think of this as a setup applicable to a country in which courts treat domestic and foreign agents differently, discriminating against foreign creditors.

The literature on international bankruptcy law documents many cases of discrimination. While explicit laws against foreigners are rare, implicit discrimination is common. For example, the principle of territorialism used in bankruptcy proceedings in most countries is prone to discrimination. Territorialism means that in the case of bankruptcy of a multinational firm, every country in which the firm owns assets opens separate bankruptcy proceedings. Bebchuk and Guzman (1999) note that in such cases courts tend to favor local creditors.\(^2\)

For simplicity, I make the extreme assumption in the model that contracts between domestic and foreign agents are not enforced, whereas contracts between domestic agents are. With capital controls, private agents are not allowed to borrow and lend abroad. Instead, governments act as sole intermediary for international capital flows. That is, they decide how much to borrow and lend and whether to repudiate international debt.

In comparing centralized and decentralized outcomes, I derive three main propositions. First, equilibria in decentralized economies look as though there was a centralized borrowing constraint on the country as a whole in the sense that, at every date, either every agent is constrained from further borrowing abroad or no agent is. This is true even though agents are heterogeneous. Second, in a decentralized economy, borrowing constraints are tighter than in one with centralized debt. That is, international borrowing and lending are reduced. Third, since constraints are tighter in a decentralized economy, centralization improves welfare. That is, when the domestic legal system gives imperfect protection to foreign creditors, there is a rationale in favor of capital controls that prohibit private agents from trading directly in international capital markets and instead gives the responsibility of international capital flows to governments.

\(^2\) In the legal literature this is also called the “grab rule.”
The paper is organized as follows: Section II presents the environment and characterizes equilibria for the decentralized economy. In Section III, I introduce capital controls of the kind mentioned above and show that this economy is equivalent to one with debt constraints as in Kehoe and Levine (1993, 1998). Section IV concludes the paper. All proofs can be found in the Appendix.

II. Model

Time is discrete and the horizon is infinite. I use a pure exchange economy with $N$ different types of agents and $M$ countries and restrict my attention to type-identical allocations. Each period there is one non-storable good. Subscripts denote the types of individuals and superscripts denote the country. Let $\lambda_i \geq 0$ denote the measure of type $i$ people in country $j$.

An event $\omega_t$ is an $N$-vector of endowments $(\omega_{it})_{i=1,...,N}$ drawn from a finite set $\Omega$. A history of events $s$ is a sequence $(\omega_1, ..., \omega_t)$ of events for periods $1, ..., t$, with $s^0$ an empty history. The probability that event history $s'$ occurs is given by $\pi(s')$. It is not necessary to put any structure on the process of shocks. In particular, I do not need to assume a Markov process.

Individuals have preferences

$$U = \sum_i \sum_{i'} \beta^t u(c_{it}(s')) \pi(s'),$$

where the period utility function $u$ satisfies the usual Inada conditions, and the time preference factor $\beta$ satisfies $0 < \beta < 1$.

In each state history there are $M + 1$ state-contingent one-period bonds, one traded internationally and one type of bond traded exclusively in each of the $M$ countries. Bond prices are $Q_{it+1}(s', \omega_{it+1})$ for international and $P_{it+1}(s', \omega_{it+1})$ for domestic bonds, and the quantities of international and domestic bonds are denoted $f_{it+1}(s', \omega_{it+1})$ and $b_{it+1}(s', \omega_{it+1})$, respectively, where $f_{it+1}(s', \omega_{it+1})$ refers to a bond traded internationally after history $s'$ that pays one unit next period if event $\omega_{it+1}$ occurs, and likewise for country $j$ domestic bonds $b_{j,t+1}(s', \omega_{jt+1})$.

After a default an individual can trade only in the domestic bond market. Hence the value after default for an individual of type $i$ in country $j$ is

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3 One can motivate the state-contingent nature with excusable default as in Grossman and Van Huyck (1988).
\[ V_t(s', b_t(s')) = \max_{d, b} \sum_{r \geq t} \sum_{s'} \beta^{r-t} u(c_t^d(s')) \pi(s'|s') \]

subject to
\[
\omega_t(s') + b_t(s') = c_t^d(s') + \sum_{s_{t+1}} P_{t+1}^t(s', \omega_{t+1}) b_{t+1}(s', \omega_{t+1})
\]
for all \( r \geq t, s' \),
\[ b_{t+1}(s', \omega_{t+1}) \geq -B \text{ for all } s', \omega_{t+1}, \]
where (2) is a no-Ponzi condition; that is, \( B > 0 \) is large enough to never bind in equilibrium.

Before a default, the consumer chooses optimal sequences for consumption and for domestic and international bonds subject to his budget and participation constraint:
\[
\max_{d, b} \sum_{r \geq t} \sum_{s'} \beta^r u(c_t^d(s')) \pi(s') \tag{CP}
\]
subject to
\[
\omega_t(s') + b_t(s') + f_t(s') = c_t^d(s') + \sum_{s_{t+1}} Q_{t+1}^t(s', \omega_{t+1}) f_{t+1}^i(s', \omega_{t+1})
\]
\[ + \sum_{s_{t+1}} P_{t+1}^t(s', \omega_{t+1}) b_{t+1}(s', \omega_{t+1}) \text{ for all } s', \]
\[
\sum_{d, s'} \sum_{s'} \beta^{r-t} u(c_t^d(s')) \pi(s'|s') \geq V_t(s', b_t(s')) \text{ for all } s', \]
\[
f_{t+1}^i(s', \omega_{t+1}), b_{t+1}(s', \omega_{t+1}) \geq -B \text{ for all } s', \omega_{t+1}, \]
and
\[
b_t^0(s') = f_t^0(s') = 0. \tag{6}
\]
The participation constraint (4) says that an individual would never want to default on international debt.\(^4\) Equation (5) is a no-Ponzi condition, where again \( B \) is large enough to ensure that the constraint never binds.

\(^4\) An alternative approach to private international debt with risk of repudiation would have been to model agents having access to international capital markets but governments making default decisions. Jeske (2001) deals with this setup.
An individual who defaults on international debt assumes that his decision does not affect domestic prices. He also assumes that he can still trade internationally indirectly, through other agents in his country. Evidently, this kind of punishment is far less severe than the autarky punishment in a Kehoe-Levine economy. In fact, at this point it is not even obvious whether this is a punishment at all. Later we will see that it is.

Let us now define and characterize equilibria in this economy.

**Definition 1.** An equilibrium is

- allocations \( \{c_i^t(s'), b_i^{t+1}(s', \omega_{t+1}), f_i^{t+1}(s', \omega_{t+1})\}_{i=1,t} \) and
- bond prices \( \{Q_{i+1}(s', \omega_{t+1}), P_i^t(s', \omega_{t+1})\}_{i=1,t} \)

such that

- each individual solves problem (CP) and
- feasibility and market clearing occur:

\[
\sum_{j=1}^{t} \sum_{i=1}^{t} \lambda^i_j c_i^j(s') = \sum_{j=1}^{t} \sum_{i=1}^{t} \lambda^i_j \omega_i^j(s') \quad \text{for all } s',
\]

\[
\sum_{j=1}^{t} \lambda^i_j b_i^{t+1}(s', \omega_{t+1}) = 0 \quad \text{for all } s', \omega_{t+1}, j,
\]

\[
\sum_{j=1}^{t} \sum_{i=1}^{t} \lambda^i_j f_i^{t+1}(s', \omega_{t+1}) = 0 \quad \text{for all } s', \omega_{t+1},
\]

Because of Walras’ law, one of the feasibility constraints is implied by the others together with consumers’ budget constraints; for completeness I include all feasibility constraints in the definition.

First-order conditions are

\[
0 = \beta' u'(c_i^t(s')) \pi(s') - \kappa^i_t(s') + \sum_{s' \leq t} \sum_{i} \mu^i_{t+s'}(s') \beta^{-1} u'(c_i^t(s')) \pi(s'|s'),
\]

(7)

\[
0 = -P_i^{t+1}(s', \omega_{t+1}) \kappa^i_t(s') + \kappa_i^{t+1}(s', \omega_{t+1})
- \mu_i^{t+1}(s', \omega_{t+1}) \frac{\partial V_i^{t+1}(s', \omega_{t+1}, b_i^{t+1}(s', \omega_{t+1}))}{\partial b_i^{t+1}(s', \omega_{t+1})},
\]

(8)

and

\[
0 = -Q_{t+1}(s', \omega_{t+1}) \kappa^i_t(s') + \kappa_i^{t+1}(s', \omega_{t+1}),
\]

(9)

where \( \kappa \) and \( \mu \) are the multipliers on the budget and participation constraints.

The next result characterizes consumption of agents with \( \mu^i_{t+s'}(s') > 0, \)
those for whom (4) holds with equality. It states that in any history in which an individual is indifferent between default and the equilibrium allocation, the consumption path after default is the same as the consumption path in the equilibrium.

**Proposition 1.** In equilibrium, if \( \mu^j_i(s') > 0 \) for some \( i, j, s' \), then \( c^j_i(s') = c^j_i(s') \) for all \( r \geq t \) and all \( s' \) such that \( \pi_i(s'|s') > 0 \), where \( c^j_i(s') \) denotes consumption after default occurred in history \( s' \).

In general, the constraint set for the consumer problem is not convex. To show that the first-order conditions for a maximum are also sufficient, I define an alternative maximization problem with the same objective function and a convex constraint set that is a superset of the original (nonconvex) constraint set. I then show in the Appendix that a solution to the alternative (convex) problem is also affordable and individually rational in the original (nonconvex) problem, which leads to the following result.

**Proposition 2.** First-order conditions for the consumer’s problem together with a transversality condition

\[
\lim_{T \to +} \beta^T \sum_{s'} u'(c^j_i(s')) \pi_i(s')[b^j_i(s') + f^j_i(s')] = 0 \quad \text{for all } i, j
\]

are also sufficient.

The next proposition characterizes bond prices.

**Proposition 3.** In equilibrium, for all \( i, j \),

\[
P^i_{r+1}(s', \omega_{r+1}) = \beta \frac{u'(c^j_{i+1}(s', \omega_{r+1}))}{u(c^j_i(s'))} \pi_{r+1}(\omega_{r+1}|s') \quad \text{for all } i, j,
\]

\[
Q^i_{r+1}(s', \omega_{r+1}) = \max_{j=1,..,M} \{P^j_{r+1}(s', \omega_{r+1})\}.
\]

In addition,

\[
Q^i_{r+1}(s', \omega_{r+1}) = P^i_{r+1}(s', \omega_{r+1}) \quad \text{for all } j, s', \omega_{r+1}
\]

implies

\[
f^j_{r+1}(s', \omega_{r+1}) = 0 \quad \text{for all } i, j, s', \omega_{r+1}.
\]

The first result says that \( P^i_{r+1}(s', \omega_{r+1}) \) is equal to the domestic marginal rate of substitution. The intuition is that, since domestic markets are complete and contracts are perfectly enforceable, marginal rates of substitution must be equal across agents within any one country. The second says that the price of the international bond is equal to the maximum of the domestic bond prices. Alternatively, the international interest rate is the minimum of the domestic interest rates. It cannot be lower, or some individuals who are not borrowing constrained would have an
arbitrage opportunity: to borrow at the low international rate and lend at a high domestic rate. It cannot be higher, or individuals in at least one country would have an arbitrage opportunity in the other direction, since nobody is ever lending constrained. Finally, the third result states that if domestic and international bond prices are equal in all states, all international borrowing must be zero.

The next result states that, given any history, within each country either everyone is borrowing constrained or no one is, even if their endowments differ.

**Proposition 4.** For all countries \( j = 1, \ldots, M \) and all histories \((s', \omega_{r+1})\), either \( \mu_{i,r+1}(s', \omega_{r+1}) > 0 \) for all \( i = 1, \ldots, N \) and \( Q_{i,r+1}(s', \omega_{r+1}) > P_{i,r+1}(s', \omega_{r+1}) \) or \( \mu_{i,r+1}(s', \omega_{r+1}) = 0 \) for all \( i = 1, \ldots, N \) and \( Q_{i,r+1}(s', \omega_{r+1}) = P_{i,r+1}(s', \omega_{r+1}) \).

What allows international borrowing and lending at all, or put differently, why do individuals repay their international debts? As propositions 3 and 4 show, the domestic interest rate is higher in exactly the periods in which the individual wants to borrow, so it is more expensive to insure against risk in the domestic market. This interest rate differential induces individuals to repay their international debts.

### III. Centralized International Borrowing and Lending

Next consider an economy in which international borrowing and lending are centralized in the following sense. The government in each country imposes capital controls of the following form: Only governments are allowed to trade internationally. Domestically, the government trades bonds with individuals in a complete market with perfect enforcement. Hence, governments act as intermediaries for international borrowing and lending. Each government’s objective is to maximize a weighted sum of its citizens’ utilities.

For simplicity I set up an economy in which governments trade internationally and allocate consumption domestically whereas agents do not access any markets. The same equilibrium allocation can be decentralized as a Ramsey equilibrium allocation in which the government borrows internationally with other governments and domestically with its citizens.

Suppose that a utilitarian government in country \( j \) has welfare weights \( \{\varphi_j^i\}_{i=1}^N \). After a default, the autarky value is the maximum weighted utility that can be attained if the whole country is banned from international borrowing and lending and the government only redistributes goods. Hence the autarky value is

\[
V_j'(s') = \max \sum_{i=1}^I \sum_{s' \in \mathcal{S}} \beta^{-u(c_j'(s'))} \pi_i(s'|s')
\]
such that, for all $s' \geq s'$,
\[
\sum_{i=1}^{I} \lambda_i \varphi_i(j, s') = \sum_{i=1}^{I} \lambda_i \omega_i(s').
\]

Before a default, the government’s problem is
\[
\max_{\varphi, j} \sum_{i=1}^{I} \varphi_i \sum_{j} \beta^j u(c_i^j(s')) \pi(s') \tag{GP}
\]
such that, for all $s'$ and $\omega_{t+1}$,
\[
\sum_{i=1}^{I} \lambda_i c_i^j(s') + \sum_{\omega_{t+1}} Q_{t+1}(s', \omega_{t+1}) f_i(s', \omega_{t+1}) = \sum_{i=1}^{I} \lambda_i \omega_i(s') + f'(s'), \tag{11}
\]
and
\[
\sum_{i=1}^{I} \varphi_i \sum_{j} \beta^{-j} u(c_i^j(s')) \pi_i(s'|s) \geq V_i(s'), \tag{12}
\]
and
\[
f_i(s', \omega_{t+1}) \geq -\bar{B}. \tag{13}
\]
That is, the government redistributes the country’s endowment plus the net borrowing and faces a participation constraint with the autarky value specified above.

**Definition 2.** An *equilibrium in the economy with capital controls* is

- an allocation \( \{c_i^j(s'), f_i(s', \omega_{t+1})\}_{i,j} \omega_{t+1} \) and
- bond prices \( \{Q_{t+1}(s', \omega_{t+1})\}_{i,j} \omega_{t+1} \)

such that

- each government solves (GP) and
- feasibility and market clearing occur:
\[
\sum_{j=1}^{L} \sum_{i=1}^{I} \lambda_i c_i^j(s') = \sum_{j=1}^{L} \sum_{i=1}^{I} \lambda_i \omega_i(s') \quad \text{for all } s',
\]
\[
\sum_{j=1}^{L} f_i(s', \omega_{t+1}) = 0 \quad \text{for all } s', \omega_{t+1}.
\]

The next proposition shows that if the utility function displays a constant elasticity of intertemporal substitution, so \( u(c) = c^{1/\theta} \), then the amount a government borrows does not depend on the welfare weights.

**Proposition 5.** If \( u(c) = c^{1/\theta} \), then \( f_i(s') \) is independent of \( \{\varphi_i\}_{i=1}^{I} \) for each \( j, s' \).

In other words, one can aggregate each country \( j \) into one represen-
tative agent with endowment $\omega_j(s') = \sum_{s=1}^{S} \lambda_j \omega_j(s')$ in each $s'$, thanks to the complete market structure within countries. Notice that there is a major difference between economies with private international debt and economies with capital controls. In an economy without capital controls, complete risk sharing can never be an equilibrium allocation, unless countries can achieve this without international borrowing and lending, as shown in proposition 3. In an economy with capital controls, however, one will observe complete risk sharing if the discount factor is high enough.\footnote{An earlier version of this paper (Jeske 2001) went through a numerical example.}

In an economy with capital controls, more international risk sharing and higher welfare are possible. The intuitive explanation is that governments can impose a more severe penalty than individuals in an economy with private international debt. For individuals, default does not affect domestic interest rates whereas for the government it does. One can think of this as the government internalizing an externality. Assume that a small open economy with private debt imposes capital controls. Then, since international bond prices did not change, the original allocation is affordable for the government and also individually rational, since participation constraints are less tight. Hence, a government can do at least as well as individuals borrowing and lending in the international market. If in addition there is a history with positive Lagrange multipliers on the participation constraint for agents in an economy with private borrowing, the government can do strictly better by relaxing the constraint in that history.

One can formalize this in the following result.

**Proposition 6.** Let $\{c_{ij}(s')\}$, $h_{ij}(s', \omega_{ij})$, $f_{ij}(s', \omega_{ij})$, be country j's equilibrium allocation in an economy with private international borrowing. Assume that $\{c_{ij}(s')\}$ solves (GP). Then

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \beta^i u(c_{ij}(s')) \pi_i(s') \geq \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{s=1}^{S} \beta^i u(c_{ij}(s')) \pi_i(s')$$

with strict inequality if there is $(s', \omega_{ij})$ with $\sum_{i=1}^{I} \lambda_j f_{ij}(s', \omega_{ij}) < 0$ (nonautarkic allocation).

### IV. Conclusion

This paper has shown that regulations on international borrowing and lending may be welfare improving. If courts do not enforce contracts between foreigners and domestic agents, then it is welfare improving to exclude agents from international markets and use governments to intermediate international capital flows.
Prohibiting private international borrowing makes people better off because a government can internalize an externality associated with international borrowing constraints. Wright (2005) builds on this paper to find a less radical way of attaining the constrained optimum through a system of subsidized private international borrowing. He performs this exercise for both the economy in the present paper with debt constraints of the style of Kehoe and Levine and Alvarez and Jermann (1998, 2000) type solvency constraints. Wright made use of the fact that default is too attractive in a world of private international borrowing. Hence, subsidizing international borrowing is another way of internalizing the externality to attain the constrained optimum.

In general, capital controls of this kind increase the amount of borrowing and lending that is possible. The government can borrow more from abroad because the participation constraint for a government is less restrictive than for an individual. In that sense this model differs from the work of Cole and English (1991, 1992), where a decrease in foreign investment was welfare improving because it lowers the probability of expropriation.

Appendix

A. Private International Debt

Drop the $i, j$ subscripts and superscripts in the derivation of first-order conditions and the proof of most propositions to simplify notation. Also write as a shortcut for $\mathbf{r}$ $\leq \mathbf{s}$ $\leq \mathbf{t}$. When first-order conditions of the consumer problem are used,

$$\kappa(s') = \beta u'(c(s'))\pi(s')[1 + \sum_{c' \leq s} \mu_c(s')\beta^{-\pi(s'|s')}]$$

Let $\eta(s')$ be the Lagrange multiplier of the $V(s', \mathbf{b}(s'))$ problem. Then

$$\frac{\partial V(s', \mathbf{b}(s'))}{\partial \mathbf{b}(s')} = \eta(s')$$

$$= u'(c^*(s'))$$

Solving for international bond prices yields

$$Q_{i+1}(s', \omega_{i+1}) = \frac{\kappa_{i+1}(s', \omega_{i+1})}{\kappa_i(s')}$$

$$= \beta \frac{u'(c_{i+1}(s', \omega_{i+1}))}{u'(c(s'))} \frac{\pi_{i+1}(\omega_{i+1})}{\pi_i(s')}$$

$$1 + \sum_{c'(\omega_{i+1})} \mu_c(s')\beta^{-\pi_{i+1}(c'(\omega_{i+1})|s')}$$

$$\times \frac{\pi(s'|s')}{\pi(s')}.$$
Hence, international bond prices are equal to the marginal rate of substitution of an individual that is not borrowing constrained this period. Solving for domestic bond prices yields

\[
P_{t+1}(s', \omega_{t+1}) = \frac{\kappa_{t+1}(s', \omega_{t+1}) - \mu_{t+1}(s', \omega_{t+1}) \frac{\partial V_t(s', \omega_{t+1})}{\partial b_{t+1}(s', \omega_{t+1})}}{\kappa(s')}
\]

\[
= \frac{\beta^{t+1} u'(c(s', \omega_{t+1})) \pi_{t+1}(s', \omega_{t+1})}{\beta u'(c(s')) \pi(s')} \times \frac{1 - A_1 + A_2}{1 + A_3}
\]

\[
= \beta \frac{u'(c(t+1)(s', \omega_{t+1}))}{u'(c(s'))} \pi_{t+1}(\omega_{t+1}|s') \times \frac{1 - A_1 + A_2}{1 + A_3},
\]

(A1)

where

\[
A_1 = \mu_{t+1}(s', \omega_{t+1}) \frac{u'(c_{t+1}(s', \omega_{t+1}))}{u'(c(s', \omega_{t+1}))} \beta^{t+1} \frac{1}{\pi_{t+1}(s', \omega_{t+1})},
\]

\[
A_2 = \sum_{t \in (0, \omega_{t+1})} \mu(s) \beta^{-1} \frac{\pi_{t+1}(s', \omega_{t+1})}{\pi(s')},
\]

\[
A_3 = \sum_{t \in (0, \omega_{t+1})} \mu(s) \beta^{-1} \frac{\pi(s'|s)}{\pi(s')}.
\]

Denote \( p_t(s) = \prod_{k=0}^{t-1} P_{s+1}(s', \omega_{t+1}) \) the implicit date 0 price of a period domestic contingent bond.

**Proposition 7.** For all \( s' \), the participation constraint (4) implies

\[
\sum_{r \leq s'} p_t(s') \sum_{s' \geq r} Q_{t+1}(s', \omega_{t+1}) f_{t+1}(s', \omega_{t+1}) \geq 0.
\]

(A2)

Moreover, if (4) holds with equality, then (A2) holds with equality.

**Proof.** For \( F \in \mathcal{R} \), define the following optimization problem:

\[
V_{t+1}^y(s', b_j(s'), F) = \max \sum_{s' \geq s} \beta^{-r} u(c_j(s')) \pi(s'|s)
\]

such that, for all \( s' \geq s' \),

\[
\sum_{r \geq s'} p_t(s') c_j(s') = \sum_{r \geq s'} p_t(s') \omega(s') + b_j(s') + F.
\]

Notice that by definition of the autarky value,

\[
V_{t+1}^y(s', b_j(s'), 0) = V_{t+1}^y(s', b_j(s'), F)
\]

and for all \( s' \), the optimal consumption path satisfies

\[
\sum_{s' \geq s} \beta^{-r} u(c_j(s')) \pi(s'|s) = V_{t+1}^y(s', b_j(s'), F(s'))
\]
if one defines

\[ F(s') = \sum_{c \in C} \beta^s u(c(s')) \pi_f(s') \]  

subject to

\[ c(s') + \sum_{\omega_{i1}} Q_{i1}(s', \omega_{i1}) f_{i1}(s', \omega_{i1}) + \sum_{\omega_{i1}} P_{i1}(s', \omega_{i1}) b_{i1}(s', \omega_{i1}) = \omega(s') + b(s') + f(s'), \]

\[ \sum_{c \in C} \beta^s \left[ f(s') - \sum_{\omega_{i1}} Q_{i1}(s', \omega_{i1}) f_{i1}(s', \omega_{i1}) \right] \geq 0, \]

\[ b_{i1}(s', \omega_{i1}) \geq -\bar{b}_i \]

\[ b_q(s^d) = f_q(s^d) = 0 \]

for all \( s', \omega_{i1} \). Let \( \kappa \) and \( \mu \) be the multipliers on the budget constraint and the alternative participation constraint, respectively. First-order conditions are

\[ \kappa(s') = \beta^s u(c(s')) \pi_f(s'), \]

\[ P_q(s', \omega_{i1}) = \frac{\kappa(s', \omega_{i1})}{\kappa(s')} - \]

\[ 0 = -Q_{i1}(s', \omega_{i1}) \kappa(s') + \kappa_{i1}(s', \omega_{i1}) + \sum_{c \in C} \mu(c(s')) P_{i1}(s', \omega_{i1}) - \sum_{c \in C} \mu(c(s')) Q_{i1}(s', \omega_{i1}). \]
Then
\[ P_{t+1}(s', \omega_{t+1}) = \beta \frac{u'(c_{t+1}(s', \omega_{t+1}))}{u'(c(s'))} \pi_{t+1}(\omega_{t+1}|s'), \]
\[ Q_{t+1}(s', \omega_{t+1}) = \kappa_{t+1}(s', \omega_{t+1}) + \sum_{r \in r(t, \omega_{t+1})} \mu_r(s') \bar{p}(s') \frac{1}{\kappa_r(s')} + \sum_{r \in r(t, \omega_{t+1})} \frac{\mu_r(s') \bar{p}(s') Q_{t+1}(s', \omega_{t+1})}{\kappa_r(s')}, \]
\[ = \beta \frac{u'(c_{t+1}(s', \omega_{t+1}))}{u'(c(s'))} \pi_{t+1}(\omega_{t+1}|s') \times \frac{1 + \sum_{r \in r(t, \omega_{t+1})} \mu_r(s') \bar{p}(s')}{\kappa_r(s')} P_{t+1}(s', \omega_{t+1}). \]
Proposition 1 yields \( c^0(s') = c(s') \) whenever the participation constraint binds. Then the first-order condition for domestic bonds in the (CP) problem is
\[ P_{t+1}(s', \omega_{t+1}) = \beta \frac{u'(c_{t+1}(s', \omega_{t+1}))}{u'(c(s'))} \pi_{t+1}(\omega_{t+1}|s') \quad \text{for all} \; t, s', \omega_{t+1}, \]
which is identical to the one in (CP'). The first-order conditions for international bonds in the two optimization problems are identical after rescaling the Lagrange multipliers on the participation constraints. Finally, by proposition 7, any allocation that solves (CP) also satisfies the participation constraint in (CP'). QED

Proof of proposition 3. As in the proof of the previous proposition, the fraction \((1 - A_1 + A_2)/(1 + A_2)\) on the right-hand side of equation (A1) collapses to one, which proves the first result. The proof of the third part is similar to the one Bulow and Rogoff (1989) use. Define
\[ g^j_i(s') = f_j^i(s') - \sum_{t = 1}^{T} f_j^{t, t+1}(s', \omega_{t+1}) Q_{t+1}(s', \omega_{t+1}) \]
and note that
\[ \sum_{t \in T} \bar{p}_t(s') g^j_i(s') \geq 0 \quad \text{for all} \; s'. \]
With complete risk sharing, \( P_i(s') = Q_i(s') \) for all \( s', j \), implies
\[ 0 \leq \sum_{t \in T} \bar{p}_t(s') g^j_i(s') = \bar{p}_t(s') f_j^i(s') \quad \text{for all} \; i, j, s', \]
where the first inequality is equation (A2), which follows from the participation constraint by proposition 7. This together with \( \sum_{t} \lambda_i f_j^i(s') = 0 \) implies \( f_j^i(s') \) for all \( s' \). QED

Proof of proposition 4. Note that the formulas for bond prices imply, for all
First-order conditions are

\[
Q_{\prime i,j}(s', \omega_{i,j}) = \frac{1 + \sum_{c_{i,j}(s', \omega_{i,j})} \mu_{i,j}(s') \beta^{-1} \pi_{\prime i,j}(s', \omega_{i,j})}{1 + \sum_{c_{i,j}(s', \omega_{i,j})} \mu_{i,j}(s') \beta^{-1} \pi(s')}, \tag{A3}
\]

Then \(\mu_{i,j}(s', \omega_{i,j}) > 0\) for one \(i\) in country \(j\) implies that \(Q_{\prime i,j}(s', \omega_{i,j}) > P_{\prime i,j}(s', \omega_{i,j})\), which in turn means that \(\mu_{i,j}(s', \omega_{i,j})\) has to be positive for all individuals in country \(j\). QED

### B. Centralized International Borrowing and Lending

Let \(\kappa\) and \(\mu\) be the multipliers on the government budget constraint and the participation constraint, respectively. I drop the country superscripts to simplify notation. First-order conditions are

\[
\phi_{i} \beta u'(c_{i}(s')) \pi(s') + \phi \sum_{c_{i}} \beta^{-1} \mu_{i}(s') u'(c_{i}(s')) \pi(s') = \kappa(s') \lambda_{i}\]

Hence,

\[
Q_{\prime i,j}(s', \omega_{i,j}) = \frac{\pi_{\prime i,j}(s', \omega_{i,j})}{\pi(s')} \beta \frac{u'(c_{i,j}(s', \omega_{i,j}))}{u'(c_{i}(s'))} \pi_{\prime i,j}(\omega_{i,j}) \frac{1 + \sum_{c_{i,j}(s', \omega_{i,j})} \mu_{i,j}(s') \beta^{-1} \pi_{\prime i,j}(s')}{1 + \sum_{c_{i,j}(s', \omega_{i,j})} \mu_{i,j}(s') \beta^{-1} \pi(s')^{-1}}
\]

and

\[
\frac{u'(c_{i}(s'))}{u'(c_{i,j}(s'))} = \frac{\lambda_{i}}{\lambda_{i,j}} \phi_{i} \phi_{j}.
\]

Alternatively, the government could have solved the planning problem in two steps.

Step 1.

\[
\max \sum_{s'} \beta u(c(s')) \pi(s') \tag{A4}
\]

such that

\[
\omega(s') + f(s') = c(s') + \sum_{c_{i,j}} Q_{\prime i,j}(s', \omega_{i,j}) f(s', \omega_{i,j}) \tag{A5}
\]

and

\[
\sum_{c_{i,j}} \beta u(c(s')) \pi(s'|s') \geq \sum_{c_{i,j}} \beta u(\omega(s')) \pi_{\prime i,j}(s'|s'), \tag{A6}
\]

where \(\omega(s') = \sum_{t=1}^{N} \lambda_{t}(s')\).
Step 2.

$$\max \sum_{i=1}^{N} \phi \sum_{j} \beta^j u(c_i(s')) \pi_j(s')$$  \hspace{1cm} (A7)$$

such that

$$\sum_{i=1}^{N} \lambda_i \epsilon(s') = c_j(s') \quad \forall s.$$  \hspace{1cm} (A8)

First-order conditions are

$$Q_{i+1}(s', \omega_{i+1}) = \beta \frac{u'(c_{i+1}(s', \omega_{i+1}))}{u'(c_i(s'))} \pi_{i+1}(\omega_{i+1}|s') \frac{1 + \sum_{s_i \in \omega_{i+1}} \mu_i(s') \beta^{i+1} \pi(s')^{-1}}{1 + \sum_{s_i \in \omega_i} \mu_i(s') \beta^{i+1} \pi(s')^{-1}}$$  \hspace{1cm} (A9)$$

and

$$\frac{u'(c(s'))}{u'(c(s'))} = \frac{\lambda_i \phi_i}{\lambda_i \phi_i} \quad \forall s, i, k.$$  \hspace{1cm} (A10)

Now one can show that the government’s problem and the proposed two-step procedure have identical solutions if the utility function displays a constant elasticity of intertemporal substitution. First derive the following result.

**Lemma 1.** If, then for all $J$, the function $h: v_{nu}(c)$ defined by is affine and strictly increasing.

**Proof.** Compute the derivative of $h$:

$$h'(x) = \sum_i \phi_i u'(\alpha_i u^{-1}(\alpha_i)) \frac{1}{u'(u^{-1}(\alpha_i))}$$

$$= \sum_i \phi_i \alpha_i \frac{u'(\alpha_i)}{u'(x)}$$

$$= \sum_i \phi_i \alpha_i.$$

Therefore, $h'(x)$ is a positive constant. QED

**Proof of proposition 5.** One has to show that the solution of the two-step procedure also solves the government’s problem. With $u(c) = c/\beta$, there is an $\alpha \in \mathcal{R}^N_{++}$ with $\sum \alpha = 1$ such that

$$c_j(s') = \alpha^i \epsilon(s') \quad \forall s', i,$$

and hence

$$\frac{u'(c(s'))}{u'(c(s'))} = \frac{u'(c(s'))}{u'(c(s'))} \quad \forall s', s', i,$$

where $\omega_i(s) = \sum_{i=1}^{N} \lambda_i \omega_i(s)$. Equations (A9) and (A10) are then just identical to first-order conditions in the original government’s problem. Also, the government’s budget constraint is satisfied when (A5) and (A8) hold. All I have to show is that the participation constraint in the government’s planning problem is satisfied. As shown in lemma 1, $h$ is affine and strictly increasing. Therefore,
Proof. \( GP \) solves (GP) and solves \( GP' \), that is, noting that the constraint set of the problem because in equation (A12), one can always set \( \gamma_i = \frac{1}{2} \).

Moreover, if solves (GP) and solves \( GP' \), then

\[
\begin{align*}
\sum_{i=1}^{l} \beta^{-i} u(c_i(s')) \pi_i(s'|s') &\geq \sum_{i=1}^{l} \beta^{-i} u(\omega_i(s')) \pi_i(s'|s') \\
\Rightarrow \left( \sum_{i=1}^{l} \beta^{-i} u(c_i(s')) \pi_i(s'|s') \right) &\geq \left( \sum_{i=1}^{l} \beta^{-i} u(\omega_i(s')) \pi_i(s'|s') \right) \\
\Rightarrow \sum_{i=1}^{l} \beta^{-i} u(c_i(s')) \pi_i(s'|s') &\geq \sum_{i=1}^{l} \beta^{-i} u(\omega_i(s')) \pi_i(s'|s') \\
\Rightarrow \sum_{i=1}^{l} \beta^{-i} u(c_i(s')) \pi_i(s'|s') &\geq \sum_{i=1}^{l} \beta^{-i} u(\omega_i(s')) \pi_i(s'|s') \\
&= \pi(s').
\end{align*}
\]

QED

Proving proposition 6 requires some more notation. Construct an altered government problem \( GP'' \) using the following participation constraint instead of (12):

\[
\sum_{i=1}^{l} \varphi_i \sum_{j=1}^{l} \beta^{-i} u(c_{ij}(s')) \pi_i(s'|s') \geq V_{j}^{\lambda}(s') \quad \text{for all } s', \quad (A11)
\]

where \( V_{j}^{\lambda}(s') \) is defined as follows:

\[
V_{j}^{\lambda}(s') = \max \left\{ \sum_{i=1}^{l} \varphi_i \sum_{j=1}^{l} \beta^{-i} u(c_{ij}(s')) \pi_i(s'|s') \right\}
\]

such that, for given bond prices \( P' \) and all \( s' \geq s' \),

\[
\sum_{i=1}^{l} \lambda c_{ij}(s') + \sum_{i=1}^{l} \lambda \sum_{j=1}^{l} P'_i(s', \omega_{j'}) h_i(s', \omega_{j'}) = \sum_{i=1}^{l} \lambda c_{ij}(s') + \sum_{i=1}^{l} \lambda h_i(s') \quad (A12)
\]

and the \( s' \) bond distribution is such that \( \sum_{i=1}^{l} \lambda h_i(s') = 0 \). One can view this as the government not taking into account the aggregate resource constraint after default and instead assuming that it can keep borrowing at domestic interest rates just like individuals in the decentralized economy. One can then derive the following result.

Lemma 2. For all \( j, \varphi_j > 0 \), and \( s' \),

\[ V_j(s') \leq V_j^{\lambda}(s'). \]

Moreover, if \( \{c_{ij}(s')\}_{ij} \) solves (GP) and \( \{c_{ij}(s')\}_{ij} \) solves (GP'), then

\[
\sum_{j=1}^{l} \varphi_j \sum_{i=1}^{l} \beta u(c_{ij}(s')) \pi_i(s'|s') \geq \sum_{i=1}^{l} \varphi_j \sum_{j=1}^{l} \beta u(c_{ij}(s')) \pi_i(s|s').
\]

Proof. The constraint set of the \( V_j \) problem is a subset of the constraint set of the \( V_j^{\lambda} \) problem because in equation (A12), one can always set \( h_i(s', \omega_{i'}) = 0 \) for all \( i, s' \), and \( \omega_{i'} \). This proves the first half of the result. Using the same argument in the other direction, that is, noting that the constraint set of (GP') is a subset of the constraint set of (GP), proves the second half. QED

Note that with the appropriate set of transfers or distribution of initial bond holdings, one can guarantee that a consumption stream in the economy with
private international debt indeed solves problem (GP'). Alternatively, one could assume that \( \varphi(s) = (\lambda/\lambda_x)|u(c^e(s))/u(c^e(s))| \) for all \( k \), in which case \( c^e(s') \) solves (GP) without transfers.

So far I proved that—keeping international interest rates fixed—under centralized borrowing a government can always do weakly better than an economy with private international borrowing. One can now prove the main result: In a nonautarkic economy the government can do strictly better.

By assumption the country as a whole borrows in history \( s' \) to pay back in \( (s', \omega_i) \). This implies that there must be a history \( (s', \omega_i) \) with \( s' \geq s' \) such that \( Q_{s}r(s', \omega_i) > P_{s}r(s', \omega_i) \) to enforce repayment of the bond, which implies \( \mu_i(s', \omega_i) > 0 \) for all \( i \) (with proposition 4) and \( f_{s}r(s', \omega_i) < 0 \) for all \( i \). This implies that from history \( (s', \omega_i) \) onward, the \( e' \) allocation must be nonautarkic; in other words, \( \sum_{i} \lambda_i c_i^e(s') = \sum_{i} \lambda c_i(s') \) for all \( s' \), where \( r \geq r + 1 \) is not possible. The reason is that the \( e' \) allocation also solves the (GP') problem, and \( f_{s}r(s', \omega_i) < 0 \) together with \( \sum_{i} \lambda_i c_i^e(s) = \sum_{i} \lambda c_i(s) \) for all \( s' \) would imply that the social planner in the (GP') problem could increase welfare by defaulting in history \( (s', \omega_i) \).

Next, recall proposition 1. The result extends to the problem (GP'); that is, with a binding participation constraint, the equilibrium \( e' \) consumption stream from \( (s', \omega_i) \) onward is identical to the stream after default in the \( V_{s}'(s', \omega_i) \) problem. Since the allocation after \( (s', \omega_i) \) is nonautarkic, the continuation values are such that \( V_{s}'(s', \omega_i) > V_{s}'(s', \omega_i) \) since the objective function is strictly concave. A social planner in the original centralized problem (GP) could therefore relax the participation constraint in history \( (s', \omega_i) \) that had a strictly positive Lagrange multiplier, and thereby increase weighted utility.

References


